Crosstalk Between Coaxial Transmission Lines

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The general theory of coaxial pairs was dealt with in an article on "The Electromagnetic Theory of Coaxial Transmission Lines and Cylindrical Shields" by S. A. Schelkunoff (*B. S. T. J.*, Oct., 1934). The present paper considers a specific aspect of the general theory, namely, crosstalk.

Formulae for the crosstalk are developed in terms of the distributed mutual impedance, the constants of the transmission lines and the terminal impedances. Some limiting cases are given special consideration. The theory is then applied to a few special types of coaxial structures studied experimentally and a close agreement is shown between the results of calculations and of laboratory measurements.

If the outer members of coaxial pairs are complicated structures rather than solid cylindrical shells, the crosstalk formulae still apply but the mutual impedances and the transmission constants which are involved in these formulae must be determined experimentally since these quantities cannot always be calculated with sufficient accuracy.

The crosstalk between coaxial pairs with solid outer conductors rapidly decreases with increasing frequency while the crosstalk between unshielded balanced pairs increases. In the low frequency range there is less crosstalk between such balanced pairs than between coaxial pairs but at high frequencies the reverse is true. The diminution of crosstalk between coaxial pairs with increasing frequency is caused by an ever increasing shielding action furnished by the outer conductors of the pairs.

Finally, crosstalk in long lines using coaxial conductors is discussed and the conclusion is reached that, unlike the case of the balanced structure, the far-end crosstalk imposes a more severe condition than the near-end crosstalk in two-way systems which involve more than two coaxial conductors.

A COAXIAL line consists of an outer conducting tube which envelops a centrally disposed inner conductor. The circuit is formed between the inner surface of the outer conductor and the outer surface of the inner conductor. Since any kind of high-frequency external interference tends to concentrate on the outer surface of the outer conductor and the transmitted current on the inner surface of the outer circuit, the outer conductor serves also as a shield, the shielding effect being more effective the higher the frequency.

Due to this very substantial shielding at high frequencies, this type of circuit has been a matter of increased interest for use as a connector

between radio transmitting or receiving apparatus and antennae, as well as a wide frequency band transmitting medium for long distance multiplex telephony or television. It has been a subject of discussion in several articles published in this country and abroad.*

The purpose of this paper is to dwell at some length on the shielding characteristics of a structure exposed to interference from a similar structure placed in close proximity. Such interference is usually referred to as crosstalk between two adjacent circuits, so that the purpose of this paper is a study of crosstalk between two coaxial circuits. In what follows we shall give an account of the theory of crosstalk, the results of experimental studies, and application of these to long lines employing coaxial conductors.

General Considerations

Let us consider a simple case of two transmission lines (Fig. 1) and let us assume that both lines are terminated in their characteristic

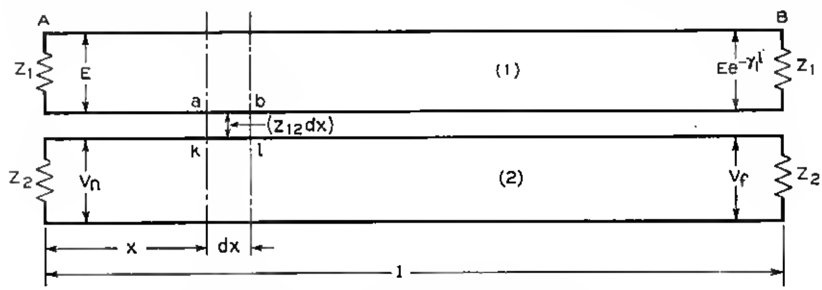

Fig. 1—Direct crosstalk between coaxial pairs.

impedances Z_1 and Z_2, and that their propagation constants per unit length are γ_1 and γ_2 respectively. If the disturbing voltage E is applied to the left end of line (1) and the induced voltage V_n is measured at the corresponding end of line (2), the ratio V_n/E is called the near-end crosstalk ratio from circuit (1) into circuit (2). Similarly, if V_f is the induced voltage as measured at the right end of line (2), when the disturbing voltage E is applied to the left end of circuit (1), we define the ratio $V_f/Ee^{-\gamma_1 l}$ as the far-end crosstalk ratio from circuit (1) into circuit (2). For convenience, we shall speak of the near-end crosstalk and the far-end crosstalk whenever the voltage crosstalk ratios are actually involved. Thus, the magnitude of crosstalk will be given by the absolute value of the corresponding crosstalk ratio. It might be expressed either in decibels as is done in this paper or it might be given

* For references see end of paper.

in terms of crosstalk units, if the absolute value of the crosstalk ratio is multiplied by a factor 10^6.

It is well to observe at this point that, depending upon special conditions, the *significant* crosstalk ratio may be either the voltage ratio or the current ratio or the power ratio. The power ratio, or more commonly the square root of it, is usually the most important but if the outputs are impressed on the grids of vacuum tubes then the voltage ratio becomes the significant measure of crosstalk. However, if one crosstalk ratio is known, any other crosstalk ratio can be readily determined provided that the characteristics of both circuits are known. Thus for the conditions of Fig. 1 the value of far-end crosstalk as given by the ratio $V_f/Ee^{-\gamma_1 l}$ in the voltage ratio system will become $(V_f/Ee^{-\gamma_1 l})(Z_1/Z_2)$ in the current ratio system.

In general, the crosstalk between any two transmission lines depends upon the existence of mutual impedances and mutual admittances between the lines. Generally, then, one can differentiate between two types of crosstalk. The first is produced by an electromotive force in series with the disturbed line in consequence of mutual impedances between the lines, and can be appropriately designated as the "impedance crosstalk." The other is due to an electromotive force in shunt with the disturbed line, induced by virtue of mutual admittances, and can be designated as the "admittance crosstalk." The two types of crosstalk are frequently referred to either as "electromagnetic crosstalk" and "electrostatic crosstalk" or as "magnetic crosstalk" and "electric crosstalk"; the latter terminology is the better of the two.

The Mutual Impedance

Consider the simplest crosstalking system consisting of two circuits only, such as shown schematically in Fig. 1. The mutual impedance between two corresponding short sections of the two lines, between the disturbing section *ab* and the disturbed section *kl*, for instance, will be defined as the ratio of the electromotive force induced in the disturbed section to the current in the disturbing section. In what follows we shall assume that the coupling between the two transmission lines is uniformly distributed; that is, that the mutual impedance between two infinitely small sections, each of length dx is $Z_{12}dx$, where Z_{12} is independent of x. The constant Z_{12} is the mutual impedance *per unit length.*

The mutual impedance between coaxial pairs will be dealt with in a later section. For the present we need only assume that this impedance can be either calculated or measured. We shall find that the crosstalk is proportional to the mutual impedance, the remaining

factors depending upon the length of transmission lines and the character of their terminations.

Direct and Indirect Crosstalk

Let us now return to the circuits shown in Fig. 1. Because of the mutual impedance between the two circuits a certain amount of the disturbing energy is transferred from line (1) to line (2), producing voltages at both ends. The voltage at the end A determines the near-end crosstalk. The type of crosstalk present in a simple system of two circuits only in consequence of the direct transmission of energy from one circuit into another we shall call the direct crosstalk. Later on we shall discuss the case where three circuits are involved in such a way that the energy transfer takes place via an intermediate circuit, causing the crosstalk which we call the indirect crosstalk. Both direct and indirect types of crosstalk have a close correspondence to the types of crosstalk used in connection with work on the open-wire lines or the balanced pairs as discussed in the paper on open-wire crosstalk.[1] The direct crosstalk of the present paper is the direct transverse crosstalk; our indirect crosstalk is the total crosstalk due to the presence of the third circuit and as such is the resultant of the indirect transverse crosstalk and the interaction crosstalk of the above paper. Following the general method outlined in the present paper one can easily subdivide the indirect crosstalk into its components. Since only simple crosstalk systems consisting of two coaxial conductors are considered in our paper, the work has not been carried through.

Direct Near-End Crosstalk

We proceed now to develop the formula for the direct near-end crosstalk. The line (1) being terminated in its characteristic impedance Z_1 the current through the generator is E/Z_1 and therefore the current in the section ab is

$$i_{ab} = \frac{Ee^{-\gamma_1 x}}{Z_1}. \tag{1}$$

Hence, by definition of the mutual impedance, the electromotive force induced in the section kl is

$$e_{kl} = i_{ab} Z_{12} dx = \frac{Ee^{-\gamma_1 x}}{Z_1} Z_{12} dx, \tag{2}$$

and the current in the section kl

$$i_{kl} = \frac{e_{kl}}{2Z_2} = \frac{Ee^{-\gamma_1 x}}{2Z_1 Z_2} Z_{12} dx. \tag{3}$$

Therefore the current at the left end of line (2) due to the electromotive force e_{kl} is given by the expression

$$(i_{kl})_n = i_{kl}e^{-\gamma_2 x} = \frac{EZ_{12}}{2Z_1Z_2}e^{-(\gamma_1+\gamma_2)x}dx. \tag{4}$$

The contribution dV_n to the potential across the left end of line (2) due to crosstalk in the section dx, x cm. away from the left end of the line, is

$$dV_n = (i_{kl})_n Z_2 = \frac{E}{2Z_1}Z_{12}\,e^{-(\gamma_1+\gamma_2)x}dx. \tag{5}$$

Hence the total induced voltage at the near end is

$$V_n = \int_0^l dV_n = \int_0^l \frac{E}{2Z_1}Z_{12}\,e^{-(\gamma_1+\gamma_2)x}dx. \tag{6}$$

Integrating, we obtain

$$V_n = E\frac{Z_{12}}{2Z_1}\frac{1 - e^{-(\gamma_1+\gamma_2)l}}{\gamma_1+\gamma_2}. \tag{7}$$

The near-end crosstalk is thus given by the expression

$$N_{12} = \left(\frac{V_n}{E}\right)_{12} = \frac{Z_{12}}{2Z_1}\frac{1 - e^{-(\gamma_1+\gamma_2)l}}{\gamma_1+\gamma_2}. \tag{8}$$

If we reversed the procedure and considered the crosstalk from circuit (2) into circuit (1), we would similarly obtain

$$N_{21} = \left(\frac{V_n}{E}\right)_{21} = \frac{Z_{21}}{2Z_2}\frac{1 - e^{-(\gamma_1+\gamma_2)l}}{\gamma_1+\gamma_2}. \tag{9}$$

By the reciprocity theorem, $Z_{21} = Z_{12}$. Incidentally, if instead of adopting as the definition of crosstalk the ratio of two voltages we regarded it as the ratio of the induced voltage to the current through the disturbing generator, we should have obtained $N_{21} = N_{12}$.

Finally, if the circuits are alike $Z_1 = Z_2 = Z_0$, $\gamma_1 = \gamma_2 = \gamma$ and the near-end crosstalk is given by the expression

$$N = \frac{V_n}{E} = \frac{Z_{12}}{2Z_0}\frac{1 - e^{-2\gamma l}}{2\gamma}. \tag{10}$$

We observe that the near-end crosstalk depends on length l. Two limiting cases are of importance here. For a length l so small, that for

a given frequency $2\gamma^2 l^2$ is negligible when compared with $2\gamma l$, we have

$$\begin{aligned} e^{-2\gamma l} &= 1 - 2\gamma l + 2\gamma^2 l^2 \\ &= 1 - 2\gamma l, \end{aligned} \tag{11}$$

and the expression (10) becomes

$$N = \frac{V_n}{E} = \frac{Z_{12}}{2Z_0} l. \tag{12}$$

The near-end crosstalk is therefore proportional to l.

For very large values of γl, that is, a very high frequency or extreme length or both, where the exponential expression is negligible as compared to unity, the expression (10) becomes

$$N = \frac{V_n}{E} = \frac{Z_{12}}{2Z_0} \frac{1}{2\gamma}, \tag{13}$$

which is independent of length.

The variation of the near-end crosstalk with length for intermediate values of γl can be best followed if instead of the expression (10) we use its absolute value

$$|N_{12}| = \left|\frac{V_n}{E}\right| = \frac{|Z_{12}|}{2|Z_1|} \frac{\sqrt{1 - 2e^{-2\alpha l} \cos(2\beta l) + e^{-4\alpha l}}}{\sqrt{\alpha^2 + \beta^2}}. \tag{14}$$

Here

$$\gamma = \alpha + i\beta, \tag{15}$$

α is the attenuation constant in nepers per unit length and β is the phase constant in radians per unit length.

We observe that for a given value of l one of the factors in (14) is oscillating with frequency. Thus, if we plot the crosstalk against frequency, the resulting curve is a wavy line superimposed upon a smooth curve, with the successive minimum points corresponding to the frequencies for which the given line is practically a multiple of half wave-lengths. The smooth curve is of course given by the magnitude of the expression (13). The curves on Fig. 2 illustrate the change of the near-end crosstalk with frequency for different lengths of a triple coaxial line made of copper conductors.

Direct Far-End Crosstalk

In order to determine the far-end crosstalk, we have to compute the induced voltage arriving at the far end of the system. Proceeding in a way similar to the derivation of the near-end crosstalk, we obtain the contribution dV_f to the potential across the right end of circuit (2), due

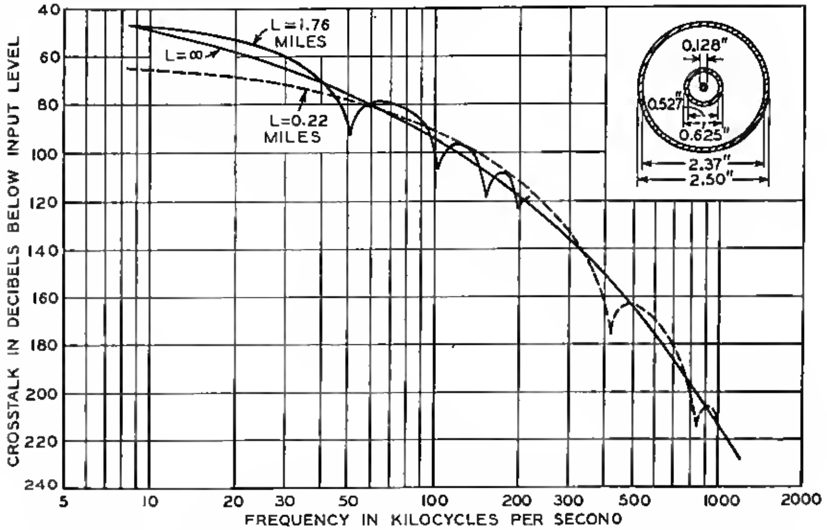

Fig. 2—Direct near-end crosstalk in a system of three coaxial conductors.

to the electromotive force in the section kl, to be given by the expression

$$dV_f = \frac{E}{2Z_1} Z_{12}\, e^{-\gamma_2 l} e^{(\gamma_2 - \gamma_1)x} dx. \tag{16}$$

Integrating this over the total length l we obtain the total voltage induced at the far end

$$V_f = E \frac{Z_{12}}{2Z_1} \frac{e^{-\gamma_2 l} - e^{-\gamma_1 l}}{\gamma_1 - \gamma_2}, \tag{17}$$

and the far-end crosstalk from circuit (1) into circuit (2) is

$$F_{12} = \frac{V_f}{Ee^{-\gamma_1 l}} = \frac{Z_{12}}{2Z_1} \frac{1 - e^{(\gamma_1 - \gamma_2) l}}{\gamma_2 - \gamma_1}. \tag{18}$$

If two similar lines are considered, with equal propagation constants and the characteristic impedances, equation (18) becomes

$$F = \frac{V_f}{Ee^{-\gamma l}} = \frac{Z_{12}}{2Z_0} l. \tag{19}$$

The far-end crosstalk is proportional to the length of the line at all frequencies.

Inasmuch as we have ignored the reaction of the induced currents upon the disturbing line, the foregoing equations must be regarded as approximations. Under practical conditions these approximations are

very good. Only equation (19) must not be pushed to its absurd implication, that for long enough transmission lines most energy will eventually travel via the disturbed line. The true limiting condition is that the energy will ultimately be divided equally between the two lines.

Crosstalk Via an Intermediate Circuit

The simplest case of the coaxial conductor system where the only crosstalk present is of the direct crosstalk type, as considered in the previous section, is the triple coaxial conductor. The mutual coupling in this case is due only to the transfer impedance between two circuits, as there are no other physical circuits involved. The case of two single coaxial conductors, the outer shells of which are in continuous electrical contact or strapped at frequent intervals, approximates the condition for the direct crosstalk if the system is sufficiently removed from any conducting matter. When two single parallel conductors in free space do not touch, an extra transmission line, an "intermediate circuit," is present consisting of the two outer shells of the coaxial conductors. Even two conductors, the shells of which are electrically connected, will form an intermediate circuit consisting of the outer shells and the other parallel conductors.

The voltage impressed on the disturbing coaxial circuit induces currents and voltages in the intermediate circuit, which now acts as a disturbing circuit for the second coaxial circuit, thus causing crosstalk. We shall now consider the near-end and far-end components of this indirect type of crosstalk.

Indirect Near-End Crosstalk

Let us consider a system shown in Fig. 3. The circuit (3) is the

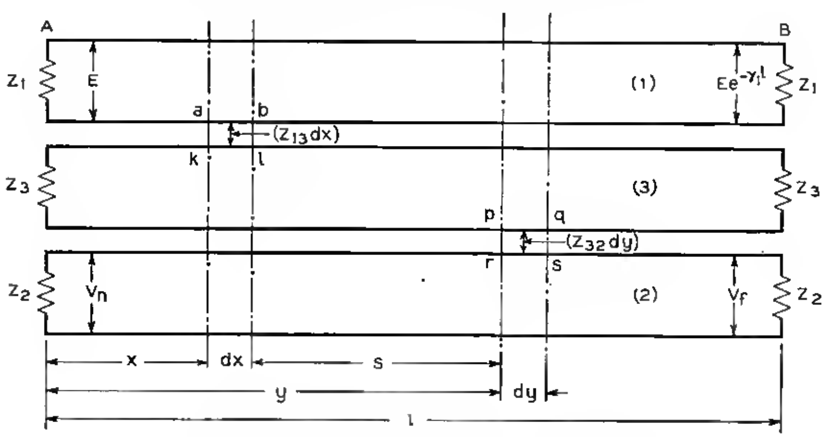

Fig. 3—Indirect crosstalk between two coaxial pairs.

intermediate circuit with an impedance Z_3 and propagation constant per unit length γ_3. Let the disturbing voltage E be applied at the A end of circuit (1). Then the current in the section kl is given by the expression similar to (3)

$$i_{kl} = \frac{EZ_{13}}{2Z_1Z_3} e^{-\gamma_1 x} dx. \tag{20}$$

The current in the section pq is

$$i_{pq} = i_{kl} e^{-\gamma_3 s}, \tag{21}$$

where

$$s = |y - x|. \tag{22}$$

The total current in the generic element of the intermediate transmission line due to coupling with circuit (1) is, then,

$$I_y = \int_0^l i_{pq} = \frac{EZ_{13}}{2Z_1Z_3} \int_0^l e^{-\gamma_3 s} e^{-\gamma_1 x} dx. \tag{23}$$

In carrying out the process of integration, we must keep in mind that from 0 to y, $s = y - x$ and from y to l, $s = x - y$.

Hence, we have

$$\int_0^l e^{-\gamma_3 s} e^{-\gamma_1 x} dx = e^{-\gamma_3 y} \int_0^y e^{(\gamma_3 - \gamma_1) x} dx + e^{\gamma_3 y} \int_y^l e^{-(\gamma_3 + \gamma_1) x} dx,$$

and

$$I_y = \frac{EZ_{13}}{2Z_1Z_3} \left[\frac{e^{-\gamma_1 y} - e^{-\gamma_3 y}}{\gamma_3 - \gamma_1} + \frac{e^{-\gamma_1 y} - e^{\gamma_3 y} e^{-(\gamma_3 + \gamma_1) l}}{\gamma_3 + \gamma_1} \right]. \tag{24}$$

The elementary electromotive force induced in the second coaxial conductor by the current I_y is

$$E_y = Z_{32} I_y dy. \tag{25}$$

The contribution of this electromotive force to the voltage across the near-end of the second coaxial pair will be then

$$dV_n = \frac{1}{2} E_y e^{-\gamma_2 y} = \frac{Z_{32}}{2} I_y e^{-\gamma_2 y} dy. \tag{26}$$

The total induced near-end voltage will be given by the expression

$$V_n = \frac{1}{2} \int_0^l Z_{32} I_y e^{-\gamma_2 y} dy. \tag{27}$$

Using the expressions (23) and (24), we obtain

$$V_n = E\frac{Z_{13}Z_{32}}{4Z_1Z_3}S_n, \tag{28}$$

where

$$S_n = \int_0^l e^{-\gamma_2 y}\left[\frac{e^{-\gamma_1 y} - e^{-\gamma_3 y}}{\gamma_3 - \gamma_1} + \frac{e^{-\gamma_1 y} - e^{\gamma_3 y}e^{-(\gamma_3+\gamma_1)l}}{\gamma_3 + \gamma_1}\right]dy. \tag{29}$$

Integrating (29) we have

$$S_n = \frac{2\gamma_3}{\gamma_1 + \gamma_2}\frac{1 - e^{-(\gamma_2+\gamma_1)l}}{\gamma_3^2 - \gamma_1^2} - \frac{1 - e^{-(\gamma_3+\gamma_2)l}}{(\gamma_3 - \gamma_1)(\gamma_3 + \gamma_2)} + \frac{1 - e^{(\gamma_3-\gamma_2)l}}{(\gamma_3 + \gamma_1)(\gamma_3 - \gamma_2)}e^{-(\gamma_3+\gamma_1)l}. \tag{30}$$

Thus, the near-end crosstalk from circuit (1) into circuit (2) via the intermediate circuit (3) is given by the expression

$$N_{12}' = \frac{V_n}{E} = \frac{Z_{13}Z_{23}}{4Z_1Z_3}S_n. \tag{31a}$$

In a similar manner we can derive the following expression for the near-end crosstalk from circuit (2) into circuit (1) via the intermediate circuit (3):

$$N_{21}' = \frac{Z_{13}Z_{23}}{4Z_2Z_3}S_n. \tag{31b}$$

The factor S_n present in (31*b*) is the same as in (31*a*), being symmetrical with respect to the subscripts 1 and 2 as a close inspection of the formula (30) would prove. $Z_{13} = Z_{31}$ and $Z_{23} = Z_{32}$ by the reciprocity theorem.

For the case of two similar coaxial pairs with equal characteristic impedances Z_0 and propagation constants γ, and symmetrically placed with respect to the intermediate line, so that $Z_{13} = Z_{32}$, we have

$$N' = \frac{(Z_{13})^2}{4Z_0Z_3}\left[\frac{\gamma_3}{\gamma}\frac{1 - e^{-2\gamma l}}{\gamma_3^2 - \gamma^2} - \frac{1 - 2e^{-(\gamma_3+\gamma)l} + e^{-2\gamma l}}{\gamma_3^2 - \gamma^2}\right]. \tag{32}$$

Now for short lengths we may use again the approximation

$$e^{-a} = 1 - a + \tfrac{1}{2}a^2. \tag{33}$$

The expression in the brackets of (32) then becomes equal to l^2 and the

near-end crosstalk is given by the expression

$$N' = \frac{(Z_{13})^2}{4Z_0Z_3}l^2, \tag{34}$$

which is proportional to the second power of length.

For γl very large we can rewrite expression (32) as follows:

$$N' = \frac{(Z_{13})^2}{4Z_0Z_3}\frac{1}{\gamma(\gamma_3 + \gamma)}. \tag{35}$$

Thus, for a system sufficiently long the near-end crosstalk via an intermediate line is independent of length.

If the intermediate transmission line is short-circuited a large number of times per wave-length, its propagation constant γ_3 becomes very large on the average and we have approximately

$$S_n = \frac{2[1 - e^{-(\gamma_2+\gamma_1)l}]}{(\gamma_1 + \gamma_2)\gamma_3}, \tag{36}$$

and

$$N_{12}' = \frac{V_n}{E} = \frac{Z_{13}Z_{23}}{2Z_1Z_3\gamma_3}\frac{1 - e^{-(\gamma_1+\gamma_2)l}}{\gamma_1 + \gamma_2}. \tag{37}$$

But $Z_3\gamma_3 = Z$, the distributed series impedance of the intermediate transmission line. Hence the "indirect" cross-talk becomes direct with the mutual impedance given by

$$Z_{12} = \frac{Z_{13}Z_{23}}{Z}.$$

Indirect Far-End Crosstalk

Using the method outlined in the previous section we arrive at the following expression for the far-end crosstalk from circuit (1) into circuit (2) via the intermediate circuit (3); see Fig. 3.

$$F_{12}' = \frac{V_f}{Ee^{-\gamma_1 l}} = \frac{Z_{13}Z_{32}}{4Z_1Z_3}S_f. \tag{38a}$$

The crosstalk from circuit (2) into circuit (1) will be given by a similar expression with Z_2 replacing Z_1 in the denominator, namely

$$F_{21}' = \frac{Z_{13}Z_{32}}{4Z_2Z_3}S_f. \tag{38b}$$

The factor S_f used in the above formulae is given by the expression

$$S_f = e^{-(\gamma_2-\gamma_1)l}\left[\frac{2\gamma_3}{\gamma_1-\gamma_2}\frac{1-e^{-(\gamma_1-\gamma_2)l}}{\gamma_3^2-\gamma_1^2} - \frac{1-e^{-(\gamma_3-\gamma_2)l}}{(\gamma_3-\gamma_1)(\gamma_3-\gamma_2)} + \frac{1-e^{(\gamma_3+\gamma_2)l}}{(\gamma_3+\gamma_1)(\gamma_3+\gamma_2)}e^{-(\gamma_3+\gamma_1)l}\right]. \quad (39)$$

When both coaxial pairs are similar and placed symmetrically with respect to the intermediate conductors we obtain the following expression for the far-end crosstalk between two coaxial conductors via an intermediate circuit:

$$F' = \frac{(Z_{13})^2}{4Z_0Z_3}\left[\frac{2\gamma_3 l}{\gamma_3^2-\gamma^2} - \frac{1-e^{-(\gamma_3-\gamma)l}}{(\gamma_3-\gamma)^2} - \frac{1-e^{-(\gamma_3+\gamma)l}}{(\gamma_3+\gamma)^2}\right]. \quad (40)$$

For small l the expression for the far-end crosstalk becomes

$$F' = \frac{(Z_{13})^2}{4Z_0Z_3}l^2, \quad (41)$$

which is the same as (34) for the near-end crosstalk.

For large l and provided the attenuation of the intermediate circuit is greater than that of the coaxial circuit we have

$$F' = \frac{(Z_{13})^2}{4Z_0Z_3}\left[\frac{2\gamma_3 l}{\gamma_3^2-\gamma^2} - \frac{2(\gamma_3^2+\gamma^2)}{(\gamma_3^2-\gamma^2)^2}\right]. \quad (42)$$

Finally, letting γ_3 approach γ and considering a limiting case when attenuation of the intermediate circuit is equal to attenuation of either of the coaxial conductors we obtain

$$F' = \frac{(Z_{13})^2}{4Z_0Z_3}\left[\frac{l}{2\gamma} + \frac{1}{2}l^2 - \frac{1-e^{-2\gamma l}}{4\gamma^2}\right]. \quad (43)$$

If the intermediate transmission line is short-circuited a large number of times per wave-length its propagation constant γ_3 becomes very large on the average. The equation (37) becomes, then,

$$S_f = \frac{2[1-e^{-(\gamma_2-\gamma_1)l}]}{(\gamma_2-\gamma_1)\gamma_3}, \quad (44)$$

and

$$F_{12}' = \frac{Z_{13}Z_{32}}{2Z_1Z_3\gamma_3}\frac{1-e^{(\gamma_1-\gamma_2)l}}{\gamma_2-\gamma_1}. \quad (45)$$

The indirect crosstalk becomes direct with the mutual impedance given by the expression

$$Z_{12} = \frac{Z_{13}Z_{23}}{Z_3\gamma_3} = \frac{Z_{13}Z_{23}}{Z}, \quad (46)$$

where $Z = Z_3\gamma_3$ is the distributed series impedance of the intermediate transmission line.

Comparison between Direct Crosstalk and Crosstalk via Intermediate Circuit for Two Parallel Coaxial Conductors

We have already seen that two parallel coaxial conductors in free space form actually three transmission circuits, the third circuit being formed by two outer shells of the coaxial conductors. When this third line is shorted by direct electrical contact or by frequent straps only direct crosstalk is present. When the third circuit is terminated in its characteristic impedance we have crosstalk via the third circuit. In this last case, however, the crosstalk via the third circuit is also the total crosstalk, since the only available path for the transfer of interfering energy is via the third circuit. Thus, we can directly compare the values of crosstalk for the system for both conditions.

We have shown that for sufficiently short lengths of the crosstalk exposure the direct type of crosstalk is given by (12) or (19), namely,

$$F = N = \frac{Z_{12}}{2Z_0} l. \tag{47}$$

We have also found that the crosstalk via an intermediate circuit is given by (34) or (41) provided that the length of conductors is small enough. Thus

$$F' = N' = \frac{Z_{13}^2}{4Z_0Z_3} l^2. \tag{48}$$

Consequently

$$\frac{F'}{F} = \frac{N'}{N} = \frac{Z_{13}^2}{2Z_{12}Z_3} l. \tag{49}$$

In seeking an experimental verification of equation (49) a series of measurements were taken on a pair of coaxial conductors of varying lengths, separations, and different terminating conditions of the third circuit. The results agreed fully with the theory.

Mutual Impedance

Like the other constants of transmission lines the distributed mutual impedance can be measured. In certain cases, however, it is possible to obtain simple formulae for this impedance. For details of such calculations the reader is referred to a paper by one of the authors.[2]

In this paper the mutual impedance is expressed in terms of *surface*

transfer impedances. Consider a coaxial pair whose outer conductor is either a homogeneous cylindrical shell or a shell consisting of coaxial homogeneous cylindrical layers of different conducting substances. The transfer impedance from the inner to the outer surface of the outer conductor is then defined as the voltage gradient on the outer surface per unit current in the conductor. In a triple coaxial conductor system this transfer impedance is evidently the mutual impedance between two transmission lines, one comprised of the two inner conductors and the other of the two outer conductors. On the other hand the mutual impedance has a quite different value if one line consists of the two inner conductors while the other is comprised of the innermost and the outermost conductors.

The surface transfer impedance of a homogeneous cylindrical shell is given by the following expression, good to a fraction of a per cent for all frequencies up to the optical range if the thickness t is smaller than 20 per cent of the average radius

$$Z_{ab} = \frac{\eta}{2\pi\sqrt{ab}} \operatorname{csch} (\sigma t). \tag{50}$$

In this equation:

a is the inner radius of the middle shell in cm.
b is the outer radius of the middle shell in cm.
t is the thickness of the middle shell in cm.
$\sigma = \sqrt{2\pi g\mu f i}$ nepers per cm.
$\eta = \frac{\sigma}{g} = \sqrt{\frac{2\pi\mu f i}{g}}$ ohms
g is the conductivity in mhos per cm.*
μ is the permeability in henries per cm.*
f is the frequency in cycles per second.

If the ratio of the diameters of the shell is not greater than 4/3 the following formula correct to 1 per cent at any frequency will hold for the absolute value of the transfer impedance

$$|Z_{ab}| = R_{DC} \frac{u}{\sqrt{\cosh u - \cos u}}, \tag{51}$$

where

R_{DC} = the dc resistance of the shell,
$u = t\sqrt{4\pi\mu g f}$.

* As in the previous paper by Schelkunoff we adhere throughout this article to the practical system of units based on the c.g.s. system. For copper of 100 per cent conductivity

$$g = 5.8005 \times 10^5 \text{ mhos/cm. and } \mu = 4\pi 10^{-9} \text{ henries/cm.}$$

The expression (51) is plotted in Fig. 3, p. 559 of Schelkunoff's paper.[2]

As it has been already mentioned, (50) and (51) represent the mutual impedance in a triple conductor *coaxial* system. One might anticipate that if the arrangement is not coaxial the mutual impedance has a different value. This is indeed the case if all three conductors have different axes. But if one transmission line is a strictly coaxial pair, then its own current remains substantially uniform around its axis and from equation (81) of Schelkunoff's paper we immediately conclude that the mutual impedance will be the same as if *all three* conductors were coaxial. *Both* transmission lines must be eccentric before their mutual impedance becomes affected by their eccentricities. Thus the mutual impedance Z_{13} between a coaxial circuit and the circuit consisting of its outer shell and a cylindrical shell parallel to it is given very accurately by (50) and (51).

The surface transfer impedance across a shell consisting of two coaxial homogeneous layers is given by

$$Z_{12} = \frac{(Z_{ab})_1 (Z_{ab})_2}{Z}, \tag{52}$$

where Z_{ab} is the transfer impedance for each layer and Z is the series impedance per unit length of the circuit consisting of the two layers insulated from each other by an infinitely thin film, when one layer is used as the return conductor for the other.

The mutual impedance between two coaxial pairs the outer conductors of which are short-circuited at frequent intervals is also given by (52) provided Z is interpreted as the distributed series impedance of the intermediate transmission line comprised of the outer shells of the given coaxial pairs. This Z is the sum of the internal impedances of the two shells $(Z_{bb})_1$ and $(Z_{bb})_2$ and of the external inductive reactance ωL_e due to the magnetic flux between the shells. If the proximity effect is disregarded, the internal impedance of a single cylindrical shell is the same as that with a coaxial return and various expressions for it are given in equations (75) and (82) in the previous paper.[2] The inclusion of the proximity effect does not complicate the formulae if the separation between the shells is fairly large by comparison with their radii, but in this case the proximity effect is not very large either. The more accurate determination of Z leads to complicated formulae; for these the reader is referred to a paper by Mrs. S. P. Mead.[6] However, at high frequencies the important factors in the mutual impedance are the transfer impedances in the numerator of (52).

Under certain conditions it is easy to obtain approximate values of the denominator of (52) and use them for gauging the limits between which the mutual impedance must lie. If the frequency is so high that the proximity effect has almost reached its ultimate value the external inductance and the internal impedance of the intermediate line are approximately

$$L_e = \frac{\mu}{2\pi}\cosh^{-1}\frac{l^2 - b_1{}^2 - b_2{}^2}{2b_1b_2},$$

$$(Z_{bb})_1 + (Z_{bb})_2 = \frac{1}{2\pi}\sqrt{\frac{i\omega\mu}{g}}\,\frac{\left(\dfrac{1}{b_1}+\dfrac{1}{b_2}\right)+\dfrac{b_1{}^2 - b_2{}^2}{l^2}\left(\dfrac{1}{b_1}-\dfrac{1}{b_2}\right)}{\sqrt{\left[1-\dfrac{(b_1+b_2)^2}{l}\right]\left[1-\dfrac{(b_1-b_2)^2}{l}\right]}}, \tag{53}$$

where b_1 and b_2 are the external radii and l is the interaxial separation. Usually $b_2 = b_1 = b$ and consequently

$$L_e = \frac{\mu}{2\pi}\cosh^{-1}\left(\frac{l^2}{2b^2} - 1\right),$$

$$(Z_{bb})_1 + (Z_{bb})_2 = \frac{1}{\pi b}\sqrt{\frac{i\omega\mu}{g}}\left[1 - 4\frac{b^2}{l^2}\right]^{-\frac{1}{2}}. \tag{54}$$

If the proximity effect is disregarded then the external inductance is simply

$$L_e = \frac{\mu}{\pi}\log_e\frac{l}{\sqrt{b_1b_2}}. \tag{55}$$

For this case, then, the mutual impedance is given by the expression

$$Z_{12} = \frac{(Z_{ab})_1(Z_{ab})_2}{(Z_{bb})_1 + (Z_{bb})_2 + \dfrac{i\omega\mu}{\pi}\log_e\dfrac{l}{\sqrt{b_1b_2}}}. \tag{56}$$

For two identical coaxial conductors the expression is further simplified to

$$Z_{12} = \frac{(Z_{ab})^2}{2Z_{bb} + \dfrac{i\omega\mu}{\pi}\log_e\dfrac{l}{b}}. \tag{57}$$

Measuring Method

As defined above, crosstalk between two transmission lines terminated in their characteristic impedances is given by the ratios of the induced and disturbing voltages. Consequently, if the input voltage into the disturbing circuit is known and the induced voltage at one of

the ends of the disturbed pair is measured, the far-end or near-end crosstalk values are obtained readily. In fact, the magnitude of the near-end crosstalk is given by the expression

$$|N| = \left|\frac{V_n}{E}\right| \tag{58}$$

and the magnitude of the far-end crosstalk is given by the expression

$$|F| = \frac{|V_f|}{|E|e^{-\alpha l}}. \tag{59}$$

Taking $20 \log_{10} \frac{1}{|N|}$ and $20 \log_{10} \frac{1}{|F|}$, we obtain an equivalent loss in db between the disturbing and disturbed levels of the two crosstalking circuits. This consideration determined the method of measurements used in our experimental studies.

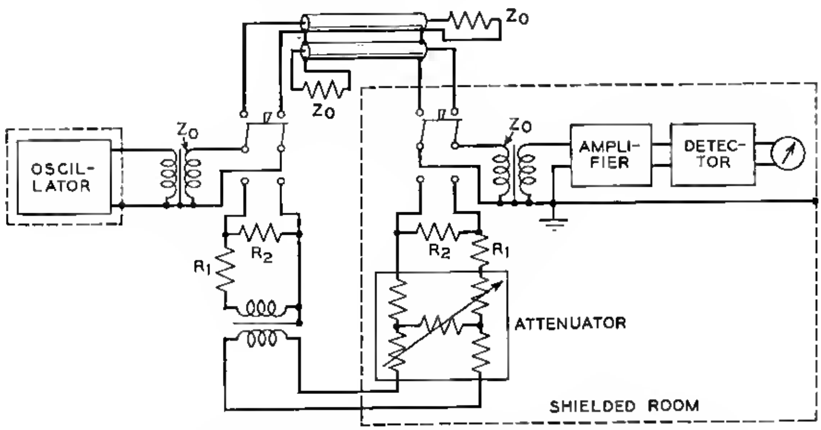

Fig. 4—Crosstalk measuring circuit arranged for far-end measurements.

The circuit used is given in Fig. 4. The two branches of the measuring set are the comparison circuits, the upper containing the crosstalking system and the lower including adjustable attenuators. The input and output impedances of both branches are kept alike by adjusting the resistances R_1 and R_2. Thus, when the lower branch of the circuit is adjusted to produce the same input into the detector as through the crosstalking branch the loss in the calibrating branch gives an equivalent crosstalk loss in db. These values of crosstalk in db below the input level in the disturbing circuit are plotted on all our sketches.

Both coaxial circuits were terminated in resistances closely equal to the absolute values of their characteristic impedances. The terminations were carefully shielded to prevent any crosstalk at these points. Careful shielding and grounding were found necessary to reduce errors due to longitudinal currents, unbalances, and interference between different parts of the measuring circuit. The overall accuracy of the measuring circuit attained was better than .5 db when the difference in input to output levels amounted to 150 db.

Agreement between Theory and Experiments

The general agreement between the theory and the experiments is indicated by the curves in Fig. 5 and Fig. 6, which give the crosstalk values for cases of two small coaxial pairs with solid outer shells in

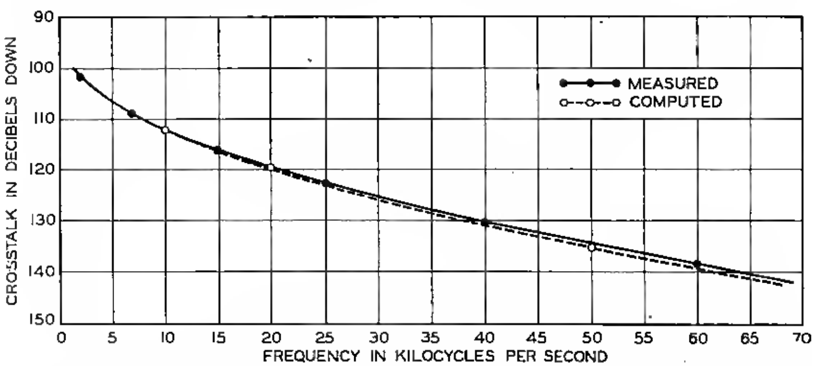

Fig. 5—Crosstalk between two coaxial pairs 20 ft. long using refrigerator pipe .032 inch thick for outer conductors. Both coaxial pairs terminated in 70 ohms. Outer conductors in contact.

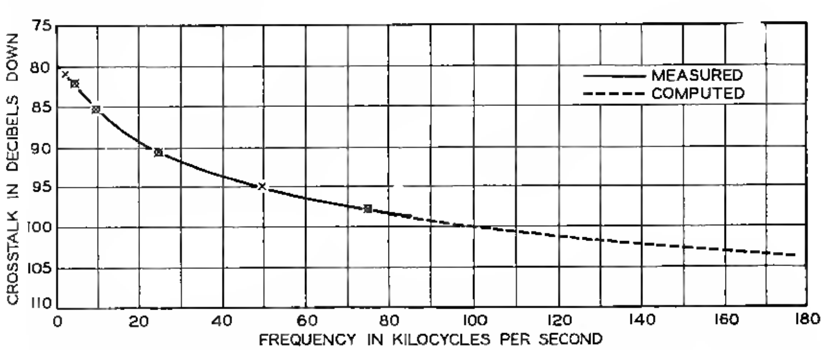

Fig. 6—Crosstalk between two coaxial pairs 25 ft. long. Outer conductor made of copper .008 inch thick, .232 inch inner diameter. Both coaxial pairs terminated in 40 ohms. Outer conductors in contact.

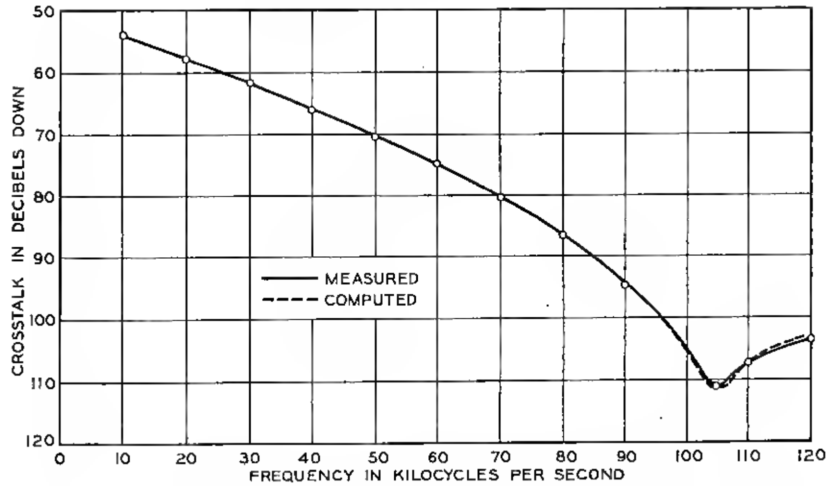

Fig. 7—Near-end crosstalk on a triple coaxial system of conductors at Phoenixville, Pa. Outer to inner circuits. Length .088 mi.

continuous contact. The curves in Fig. 7 show a comparison between measured and computed values of near-end crosstalk for a system of three coaxial conductors .88 mile long as installed at Phoenixville, Pennsylvania.

Also, as was already stated above, full agreement between theory and experiments was established as to validity of equation (49).

Crosstalk in Long Lines Employing Coaxial Conductors

In a system consisting of two coaxial pairs, where two outer conductors are in contact, essentially only one kind of crosstalk is present depending on the direction of transmission on both pairs. It is near-end crosstalk when transmitting in opposite directions and far-end crosstalk for transmission in the same direction. Where more than two coaxial conductors are grouped together and transmission is in both directions both types of crosstalk are present.

Although for a sufficiently short length of crosstalk exposure near-end and far-end crosstalk are identical, in a sufficiently long system the transmission characteristics of the line and associated repeaters will make a marked difference between them. It has been a common experience that in a long system using unshielded balanced structures near-end crosstalk imposes more severe requirements on balance between crosstalking circuits than far-end crosstalk.

We shall now consider a coaxial pair. Here, the magnitude of the far-end crosstalk was found to be given by expression (19). The

magnitude of the near-end crosstalk is given by expression (14), which for equal level points becomes

$$|N| = \left|\frac{Z_{12}}{2Z_0}\right| \frac{e^{\alpha l}\sqrt{1 - 2e^{-2\alpha l}\cos(2\beta l) + e^{-4\alpha l}}}{2\sqrt{\alpha^2 + \beta^2}}. \tag{60}$$

Thus, the ratio of the corrected near-end to the far-end crosstalk is obtained by combining equations (60) and (19):

$$\left|\frac{N}{F}\right| = \frac{e^{\alpha l}\sqrt{1 - 2e^{-2\alpha l}\cos(2\beta l) + e^{-4\alpha l}}}{2\sqrt{(\alpha l)^2 + (\beta l)^2}}. \tag{61}$$

The curve in Fig. 8 gives the db difference between near-end and far-end crosstalk for different frequencies on a 10-mile length of two

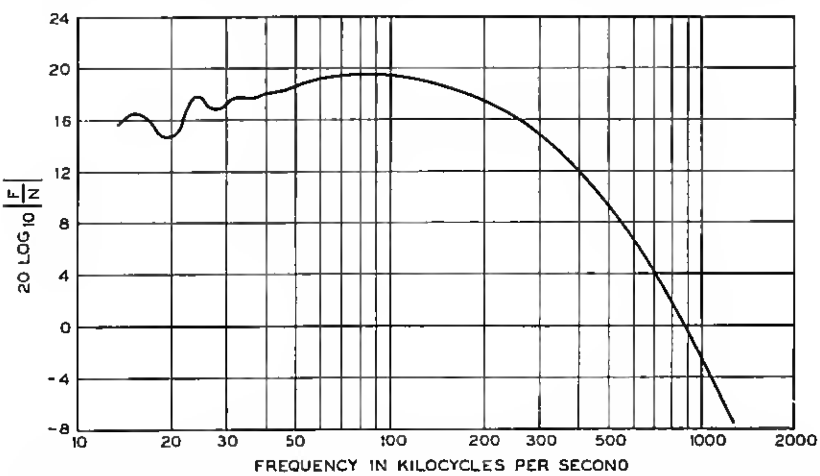

Fig. 8—Values of 20 $\log_{10} |F/N|$ for a 10 mi. repeater section of two parallel coaxial pairs in continuous contact. Coaxial pairs consist of No. 13 AWG solid copper wire, .267 in. inner diameter copper outer conductor .020 in. thick, and rubber disc insulation.

parallel coaxial pairs with hard rubber disc insulation. Each pair consists of a copper outer conductor of .267″ inner diameter and .020″ thick, and a .072″ solid copper inner conductor. It is evident that in a single repeater section far-end crosstalk is higher than near-end crosstalk up to about 900 kc.

When a number of repeater sections are connected in tandem the near-end crosstalk contribution from a single repeater section will reach the terminal of the system modified both in magnitude and in phase due to transmission through intervening sections of crosstalking circuits. At the terminal the phase changes will distribute the crosstalk from all sections in a random manner, which, in accord with both the theory and

experimental evidences, will result in a root-mean-square law of addition. Thus, the overall near-end crosstalk from m sections will be equal to the crosstalk from a single section multiplied by the square root of m.

On the contrary, in a system using similar coaxial pairs transmitting in the same direction and employing repeaters at the same points, the far-end crosstalk is affected mostly by the phase differences of the repeaters. If these do not vary from the average by more than a few degrees, the far-end crosstalk in a system involving even a comparatively large number of repeaters will change proportionally to the first power of the number of repeater sections m. Only with a very large number of repeater sections (perhaps 500 or more) and random phase differences of repeaters and line of perhaps 5°–10° will the far-end crosstalk from single sections tend to approach random distribution. In this case the root-mean-square law will hold reasonably well.

Thus, far-end crosstalk will grow faster than near-end crosstalk as the number of repeater sections increases. This, combined with the relationship between the far-end and the near-end crosstalk in a single repeater section as given by equation (61) and Fig. 8, leads us to conclude that in long systems with both near- and far-end crosstalk present the limiting factor will be the far-end crosstalk. This is contrary to the experience with balanced structures stated above.

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